JOINT UNIVERSITIES PRELIMINARY EXAMINATIONS BOARD

2015 EXAMINATIONS

MATHEMATICS: SCI-J154

MULTIPLE CHOICE QUESTIONS

1. Find the non-zero negative value of x which satisfies the equation

$$\begin{vmatrix} x & 1 & 0 \\ 1 & x & 1 \\ 0 & 1 & x \end{vmatrix} = 0$$

A. 2

B. -2

C. $\sqrt{2}$

D. $-\sqrt{2}$

2. *If* $Z = \begin{bmatrix} 2 & 3 & 3 \\ 4 & 1 & 0 \\ 1 & 4 & 0 \end{bmatrix}$, find determinant of Z.

A. 35

B. 45

C. -35

D. 48

3. Compute $\left(1 + \frac{3}{1+i}\right)^2$.

A. $\frac{-8}{2} - \frac{15}{2}i$

B. $4 - \frac{15}{2}i$

C. $\frac{17}{2} - \frac{15}{2}i$

D. 4

4. Find the centre and radius of the circle $8x^2+8y^2-24x-40y+18=0$.
   A. (3/2, 5/2) and r = 3/2
   B. (-3/2, 5/2) and r = 5/2
   C. (3/2, -5/2) and r = 3/2
   D. (3/2, 5/2) and r = 5/2

5. Find the equation of the tangent to the circle $2x^2 + 2y^2 = 30$ at the point $(-3, 6)$.
   A. x + y – 15=0
   B. x – 2y + 5=0
   C. x + 2y – 5=0
   D. x – 2y+15=0

6. Given the equations of the ellipse $x^2/2+y^2=1$. Find the equation of the directrices.
   A. $x = (0, \pm 1)$
   B. $x = (0, \pm 2)$
   C. $x = (0, \pm 3)$
   D. $x = (0, \pm 4)$

7. Find the gradient of the curve $y = x^3 - 6x^2 + 11x - 6$ at the point (1, 0)
   A. -1
   B. -2
   C. 1
   D. 2

8. Given sets $A = \{a, b, 1, 3\}$ and $B = \{a, 2, 4\}$, find $A \cup B$.
   A. $\emptyset$
   B. $\{a, b, 1,2,3,4\}$
   C. $\{a, b, 1,3\}$
   D. $\{b, 1,2,3,4\}$

9. Let P be the set of prime factors of 42 and Q be the set of prime factors of 45. Find $P \cap Q$.
   A. {2}
   B. {3}
   C. {7}
   D. {5}

10. A polynomial $2x^3 + ax^2 + bx - 1$ has a factor $(x-1)$ and the remainder when it is divided by $(x-2)$ is -4. Find $a+b$.

A. -1

B. 1

C. -2

D. 2

11. Solve the equation $\log_3 x + \log_x 3 = \frac{10}{3}$

A. $\sqrt{3}$, 9

B. 27 , $\sqrt{3}$

C. 10 , 9

D. 27 , $\sqrt[3]{3}$

12. Solve the equation $\sqrt{2x+3} - \sqrt{(x-2)} = 2$

A. 3 , 6

B. 3 , 11

C. 27 , 3

D. 3 , 10.

13. If $y = x\,(x^6 - 1)$, find the range for which $y = 0$.

A. $(-\infty, 0) \cup (0, \infty)$

B. $(-1, -\infty) \cup (0, \infty)$

C. $[-1, 0) \cup [0, 1]$

D. $(-\infty, \infty)$

14. Evaluate $\lim_{x \to -3} \left\{\frac{3x^2 - 27}{x+35}\right\}$

A. -18

B. 9

C. 0

D. 3

15. Evaluate $\int_1^e \frac{1}{x} dx$

A. 0

B. 2

C. 1

D. 2e

16. Evaluate $\int_0^{\frac{\pi}{2}} Cos\ x\ dx$

A. 2

B. 7

C. -1

D. 1

17. Evaluate $\lim_{x \to \infty} \left\{ \frac{2x^3 + x^2 - 5}{x^3 + 2x + 1} \right\}$

A. 5

B. 0

C. 2

D. ∞

18. The expression $px^2 + qx + r \ equals\ 4\ at\ x = 1$. If the derivative is 2x + 1, what are the values of p, q and r respectively

A. 1, 1, 2

B. 1, 2, 1

C. 1, 0, 1

D. 1, -1, 2

19. The gradient of a curve at any point (x, y) is given by 2x + 3. If the curve passes through the origin, find the equation of the curve

A. x (x + 2)

B. x (2x + 3)

C. $x^2 - 4$

D. 2x + 3

20. The position of an object in motion at any time (t) is given by s = $3t^3$ - 5t – 2. Obtain the velocity of the object after 2 seconds.
A. 31m/s

B. 36m/s

C. 18m/s

D. 20m/s

21. Find the derivative of $2x^3 – 5x^2 + 2$
A. $x^2 – 10x$

B. $6x^2 – 10x$

C. $–10x – 6x^2$

D. 6x – 10.

22. Find the derivative of y = (3 + 2x) (1 – x)
A. -1 - 4x

B. 4x -1

C. -4x + 1

D. – 4x

23. Differentiate $(x +y)^2 = 5$.
A. – 4

B. – 2

C. – 1

D. 10

24. Evaluate: $\lim_{x\to 5} \frac{x^2 -25}{x-5}$
A. 5

B. 15

C. 10

D. 12

25. If $y = (x - 1)\, e^{-x}$, find $\frac{dy}{dx}$

A. $(2 - x)\, e^{-x}$

B. $e^{x}\, 2x$

C. $- x\, e^{x}$

D. $2x$

26. Find the modulus of 2i + 3j – 4k

A. $\sqrt{12}$

B. $\sqrt{29}$

C. $\sqrt{3}$

D. $\sqrt{28}$

27. Find the scalar products of a = 2i + 3j and b = - i + 4j

A. 20

B. 10

C. -10

D. -20

28. Find the value of n for which the vector si +nj -3k and ni – j + 5k are perpendicular.

A. 90

B. 0^0

C. $\frac{15}{s-1}$

D. $\frac{s-1}{15}$

29. Obtain the projection of vector a = (3,-1.5) on the vector b = (2.1,-3)

A. $\frac{-2}{\sqrt{14}}$,

B. $\frac{-2}{\sqrt{35}}$

C. $\sqrt{14}$

D. $\sqrt{35}$

30. Find the volume of the tetrahedron OABC with point A (2,1,1) ,B(0,-1,1) and C(-1,3,0).

A. $^2/_5$

B. $^3/_4$

C. $^4/_3$

D. $-^4/_3$

31. The distance S in meters (m) moved by a particle in t time in seconds (s) is given by $S = 1.5t^2 - t$. Find its speed after t seconds.

A. 3t m/s

B. (3t-1)m/s

C. (3t+1)m/s

D. (1-3t)m/s

32. A car starts from A and travels 10km due West, 20km North-West and 30km due North. Find the displacement from A.

A. 51.3km

B. 53.3km

C. 43km

D. 50.3km

33. The brakes of a train are able to produce a retardation of 1.2m/s. if the train is travelling at 90km/h, at what distance from a station should the brakes be applied.

A. 200m

B. 250m

C. 260m

D. 240m

34. A particle is projected with a velocity of 20m/s up a smooth inclined plane of inclination 30°. Find the distance described up the plane.

A. 40.8m

B. 48m

C. 40m

D. 38m

35. A block of mass 20kg rests on a horizontal plane whose coefficient of friction is 0.4. Find the least force required to move the block if it acts horizontally.

A. 190N

B. 80N

C. 196N

D. 78.4N

36. A mass of 8kg hangs in equilibrium, suspended by two light inelastic strings making angles 30° and 45° with the horizontal, calculate the tensions in the two strings.

A. 57.4N, 70.3W

B. 50N, 70W

C 60.5N, 60.5W

D. 50N, 50W

37. If $\vec{a} = 2i + 3j + 5k, \vec{b} = 3i - 5j + 2k,\ \vec{c} = i - j.$ calculate λ such that $2\vec{a}$- $5\vec{b} + \lambda\vec{c}$ is perpendicular to the $x - axis.$

A. 8

B. 9

C. 10

D. 7

38. The probabilities that John and Joanna will passed an examination are $\frac{2}{3}$ and $\frac{4}{5}$ respectively.

Find the probability that only one of them will pass.

A. $\frac{2}{15}$
B. $\frac{4}{15}$
C. $\frac{1}{15}$
D. $\frac{6}{15}$

39. In how many ways can a committee of 2 men and 2 women be formed from 3 men and 5 women?

A. 12

B. 30

C. 20

D. 10

40. The formular for Spearman's rank correlation is:

A. $1 + \frac{6\sum\ d^2}{N(N^2-1)}$

B. $1 - \frac{\sum\ d^2}{N(N^2-1)}$

C. $1 - \frac{6\sum\ d^2}{N(N^2-1)}$

D. $1 - \frac{6\sum\ d^2}{N^2}$

41. The following are continuous random variables except

A. The temperature of an object

B. The distance between two points

C. The population of a school

D. The marks obtained by a group students

42. The following are features of a standard normal curve except

A. It is bell-shaped

B. The area under the curve is 1

C. It is symmetric about the mean

D. The variance is zero

43. An experiment in which the outcomes are two possibilities: "Success" or "failure" is said to be

A. Binomial

B. Normal

C Geometric

D. Bernoulli

44. The range of values of rank correlation ($r_{rank})$ is

A. $-1 \leq r_{rank} \leq 1$

B. $0 \leq r_{rank} \leq 1$

C. $-1 \leq r_{rank} \leq 0$

D. $r_{rank} \geq 1$

45. Find the geometric mean of the data: 5, 15, 10, 8, 12.

A. 72000

B. 821.1

C. 9.36

D. 10

46. One can easily determine the ... of a distribution from histogram.

A. mean

B. mode

C. median

D. standard deviation.

47. Find the mean of the following scores

Scores(x)	61	64	67	70	73
Freq. (f)	5	18	42	27	8

A. 65

B. 67.45

C. 67

D. 68

48. What is the mode of the following numbers 1,8,8,10,9,2,7,8,2,2,4,1,1,8,7,1

A. 8

B. 8 and 1

C. 1

D. None of the above

49. The .... level of a test is the maximum probability of committing Type I error when the null hypothesis holds.

A. acceptance

B. rejection

C. significance

D. significant

50. The standard deviation of a statistic describes

A. the shape of its distribution.

B. the centre of its distribution.

C. the amount of skewness associated with its distribution.

D. the amount of variability associated with its distribution.

MATHEMATICS ESSAY QUESTIONS

1 (a). Given A = {-5, -3, -1, 0, 1, 2, 3} , B = {-4, -3, 0, 3, 5, 8}. **MAT001**

Find A Δ B. 2 Marks

(b) If A, B, and C are any sets, show that $A \cup (B \cup C) = (A \cup B) \cup C$ 3 Marks

(c) In an election involving three parties for the chairmanship and gubernatorial election of Lagos State, voters cast their votes as follows:

190 voted for party A, 200 for party B and 250 for party C. 80 voted for A and B, 60 voted for A and C, 100 voted for B and C and 40 voted for B alone.

If 500 people voted during the election, find:

i. The number of voters who voted for all the three parties. 3 Marks

ii. The number of voters who voted for A and B but not C. 3 Marks

iii. The number of voters who did not vote for any party. 4 Marks

2 (a) i. Evaluate the determinant A. **MAT 001**

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$ 3 Marks

ii. what do you conclude from 2a(i)? 1 Mark

iii. Resolve $\frac{x^3-1}{(x+3)(x+1)^2}$ in partial fractions. Hence, obtain its Binomial expansion up to terms x^2. 4 Marks

(b) If $\cos(x + \alpha) = \sin(x + \beta)$, find $\tan x$ in terms of α and β. 3 Marks

(c) If $\sin A = \frac{4}{5}$ and $\cos B = \frac{12}{13}$, where A is obtuse and B is acute, find without using tables the values of: i. $\sin(A + B)$ ii. $\tan(A - B)$. 4 Marks

3 (a) By using the reduction formula for $\int \sec^n x dx$, evaluate the definite integral **MAT 002**

$$\int_0^{\frac{\pi}{4}} \sec^6 x dx$$ 10 Marks

(b) Find the area enclosed by the curve $y = x^2$ and the x-axis between

$x_1 = 0$ and $x_2 = 2$. 5 Marks

4 (a) Evaluate $\lim_{x \to 0} \left(\frac{1}{x} - \csc x\right)$. **MAT 002**

3 Marks

(b) By Taylor's theorem, show that $\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \frac{x^n}{n}$, and

hence evaluate $\log_e(1.1)$ to four decimal places. 8 Marks

(c) Using the trapezoidal rule with ordinates x = 1.0, 1.4, 1.8, 2.2, 2.6, 3.0;

evaluate $\int_1^3 \frac{1}{x+1} dx$. 4 Marks

5. In the study of motion of rigid bodies, explain the following concepts: **MAT 003**

(a) i. Moment of inertia of the system 4 Marks

ii. Radius of gyration of the system. 4 Marks

(b) Find the moment of inertia and radius of gyration of a uniform thin rod of length 2a, density ρ about an axis passing through one end of the rod perpendicular to its length 7 Marks

6 (a) State the Newton's law of cooling and write out the differential equation describing the temperature of the body. **MAT 003** 4 Marks

(b) A beaker of water initially at 100^0C is allowed to cool in a room maintained at 15^0c. After two minutes, the water temperature is 85^0C. Find the temperature of the water after four minutes and the time taken for the water to reach 40^0C (Hint: use Newton's law of cooling 6(a) above). 5 Marks

(c) If the position vectors of points A, B and C are $\underline{a} = \underline{i} + 3\underline{j} - 7\underline{k}$, $\underline{b} = 7\underline{i} + 6\underline{j} + 5\underline{k}$ and $\underline{c} = 9\underline{i} + 7\underline{j} + \beta\underline{k}$, respectively. Find

i. $|\underline{a} + \underline{b}|$ 3 Marks

ii. the value of β if A, B and C are Collinear. 3 Marks

7(a) The following data represent scores of 50 students in a Statistics test. **MAT 004**

72 93 70 59 78 74 65 73 80 57 67 72 57 83 76 74 56 68 67 74 76

79 72 61 72 73 76 67 49 71 53 67 65 100 83 69 61 72 68 65 51 75

68 75 66 77 61 64 74 72

By using a class interval of five (45 – 49, 50 – 54, etc):

i. Prepare the frequency distribution table. 4 Marks

ii. What is the coefficient of variation? 4 Marks

iii. Does the data represent a sample or a population? 1 Mark

(b) Discuss briefly the measures of location associated with frequencies hence; explain mean, mode, and median. 6 Marks

8(a) i. Find the coefficient of linear correlation between the variables A and B in the below table **MAT 004** 3 Marks

A	1	2	3	4	5
B	1	2	3	6	8

ii. Five students were ranked according to their scores in Mathematics and Physics thus:

Student	**A**	**B**	**C**	**D**	**E**
Mathematics	1	3	5	2	4
Physics	2	1	3	4	5

Calculate the Spearman's rank correlation coefficient. 3 Marks

(b) Differentiate between discrete and continuous random variable. 2 Marks

(c) A company that manufactures computer chips, finds that 5% of the chips they produce are defective. If 8 chips are selected at random, find the probability that:

i. 2 chips will be defect 2 Marks

ii. at least 2 chips will be defective. 2 Marks

iii. calculate for (i) and (ii) above, the number of expected defective chips and variance in a sample of 2, 000 chips. 3Marks