ADVANCED MATHEMATICS (A12) SYLLABUS

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INTRODUCTION:

This course is designed to provide the candidates an opportunity to further their education into higher level education via direct entry mode. It will aid in the conceptualization, interpretation and application of mathematical problem and also enhance their solving ability in their profession of study.

OBJECTIVE:

The general objective of the course is to give candidates an opportunity to further their education into higher level of education such as universities, polytechnics etc.

EXAMINATION SCHEME

The examination consists of two papers of three hours each.

The total mark for each paper is 100.

Each paper shall consist of three (3) sections, namely: A, B and C.

Section A: This section consists of Seven (7) questions for candidates. They are required to answer five (5) questions from this section. The section carries 20 marks, Each question carries 4 marks. Time required for this section is (40) minutes i.e. an average of 8 minutes to a question.

Section B: This section consists of six (6) questions for candidates. Candidates are required to answer four (4) questions only. The section carries 40 marks with each question carrying 10 marks. The required time is 1 hour 10 minutes.

Section C: The same as in section B.

NOTE:

Questions in Sections B and C should be of higher difficulty level than those in Section A.

EXAMINATION MATERIALS

Candidates are allowed to use the recommended mathematics and statistical tables in the examination hall for the paper. It is strongly recommended that candidates obtain copies of these tables for use throughout the course.

Candidates should bring rulers and complete mathematical instruments set for the paper. Borrowing of instrument from other candidates in the examination hall shall not be allowed. The use of noiseless, cordless and non-programmable calculators is allowed.

PAPER 1

S/N	TOPICS/OBJECTIVES	CONTENTS	ACTIVITIES/REMARKS
1.	**Ratio and proportion** Solve problems on ratio and proportion.	Ratio and proportion. Relationship between ratio and proportion, representative fraction. Examples and exercises on direct and inverse ratios and proportions including representative fraction.	Relate these to the students' work in science, technical and business subjects.
2.	**Equation** a. Simple (Linear) Equations: Solve problems involving simple equations	Simple Equations: Illustrate the meaning of equity with reference to simple equations by using the idea of simple balance. Bring out the meaning of equality sign by adding or subtracting same quantities to each side or by multiplying and dividing each side by a common factor (excluding zero). Solving of simple equations e.g. $2x + 4 = 5x - 6$, $3x = 6 + x$ etc. Solving equations with fractional coefficient e.g. $^1/_3 (x + 4) + {}^3/_3 (2x + 6)$ etc. Solve word problems involving linear equations.	The expression 'cancel out' should be avoided. Students to build linear equations with the teacher's guidance.
	Simultaneous Linear Equation: b. Identify, formulate and solve simultaneous linear equations in two and three variables. Solve word problems involving simultaneous equations in two and three variables.	Linear equations in two variables. Linear equation in three variables; including word problems.	Relate the equations to Technical and Business activities
	c. **Quadratic Equation:** i. Solve quadratic equations using appropriate methods. ii. Construct quadratic equations with given roots. iii. Solve word problems involving quadratic equations.	Definition of quadratic equations. Solving of quadratic equation by factorization. Solution by completing the square. Given an expression of the form $y = x^2 \pm ax$, trainers should be able to find a constant k to which can be added to make the expression a perfect square. Deduce the formula of quadratic equation from completing the square. Solve quadratic equations using the formula.	Difference between an equation and expression should be emphasized.
3.	**Percentages, Profit and Loss**	Percentages, profit and loss calculation. Conversion of fractions and decimals to percentages and vice versa; percentage	The means of transactions e.g. money, cheques, money orders, postal orders etc should be

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	Apply the principles of percentage to fractions and decimals	change, commercial arithmetic including profit and loss, small decimal fractions. Application of profit and loss to Commerce generally.	mentioned.
4.	**Surds:** Simplify and rationalize surds including fractional surdic expressions	Simplification and Rationalisation of simple surds	Surds of the form $a/\sqrt{b}$ and $a\sqrt{b}$ where a is a rational number and b is a positive integer.
5.	**Indices and Logarithm** a: **Indices:** Apply the laws of indices in simplification and calculations. Solve simple indicial equations.	Indices as a shorthand notation. Laws of Indices $a^x \times a^y = a^{(x+y)}$; $a^x \div a^y = a^{(x-y)}$, etc Equations involving indices.	The use of indices in science, technical and commercial subjects should be emphasized and exercises should be related to practical use. Trainers should be encouraged to discover the laws and deduce the meaning of a^0, a^{-n}, etc.
	b: **Logarithm** Apply Logarithm, square and square root tables in calculations. Apply the basic rules of logarithms.	Base 10 logarithm tables and anti-logarithm tables, calculation involving multiplication, division, powers and roots using logarithm tables. Examples and exercises from simple to complex, combination of multiplication, division, powers and roots of numbers e.g. $\frac{\sqrt{271.4} \times 28.6^3}{\sqrt[3]{13.4}}$ Rules of logarithm: $\log_a(xy) = \log_a x + \log_a y$ $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$ $\log_a x^p = p\log_a x$ Sketches and comparison with indices to be made. Copious examples to lead to the verification of these rules. For example, $\log_{10}100 = \log_{10}(25 \times 4)$	

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		$= \log_{10} 25 + \log_{10} 4$,etc.	
	c: **Relationship between indices and logarithm:** Explain the relationship between indices and logarithms.	Indices and logarithm as inverse operations: e.g. $y = 10^x \Leftrightarrow x = \log_{10} y$ $\log_a x = y \Leftrightarrow a^y = x$	
6.	**Arithmetic and Geometric Progression** Identify sequence patterns and calculate the nth term of a given sequence in AP and GP. Calculate the sum of AP and GP.	Sequences and series. Difference between AP and GP. nth terms of AP and GP. Sum of AP and GP.	
7.	**Variation**	Direct, inverse, joint and partial variations.	Application to simple practical problems in Technical and Business situations.
8.	**Linear Inequalities:** Solve simple linear inequalities in one variable. Find solution set of two or more linear inequalities in two variables. Represent solution of linear inequalities on number line.	Solution of linear inequalities in one variable and representation on number line. Graphical solution of linear inequalities in two variables.	Simple practical application to life problems.
9.	**Simultaneous Linear Equation.** Identify, formulate and solve problems involving linear equations in three unknown values.	Linear equations in three unknown values.	
10.	**Sets, Relations, Functions and Operations** The definition of a set, finite and infinite sets, equality of sets, subsets, union, intersection, universal set, complements, and empty sets. Venn diagram, symmetric difference, power sets and De Morgan's law. Inclusion – exclusion principle. Elements of relations functions and operations.	Construct elementary examples of functions. Also construct simple examples to elucidate inclusion – exclusion principle. De Morgan's laws need to be proved analytically. Proofs by Venn diagrams are acceptable.	

11.	**Algebra and Inequalities.** Definition of absolute value for modulus of a real number. Solving inequalities involving linear and quadratic functions. Solution sets of inequalities.	Divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero). Use the Factor theorem and the Remainder theorem, e.g. to find factors, solve polynomial equations or evaluate unknown coefficients. Construct specific examples of inequalities such as $ax + b > 0$, $ax^2 + bx + c > 0$, $\frac{x-a}{x-b} > 1$ and solve them.	
12.	**Principle of Mathematical Induction and its Applications** Intuitive definition of sequence and a series. Arithmetic and Geometric progression. Arithmetic and Geometric means. The sigma notation, evaluation of $\sum n$, $\sum n^2$ by using mathematical induction. Apply the AP and GP formulae to find any term and sum of the finite and infinite series. Calculate the mean, common difference, and common ratio of the series.	Sequences and series. Difference between AP, and GP. The nth term and the sum of the finite and infinite series. Also evaluate Arithmetic and Geometric means and know their relationship. Only finite cases need to be treated except in geometric progression where common ratio is less than one. Mean, Common difference and common ratio of the series. These topics should not be limited to the NBC/NTC Mathematics.	Scope and depth of treatment of these topics should not be limited to NBC/NTC.
13.	**Quadratic and other Polynomial Functions** Elementary properties of quadratic expressions. Sums and products of roots of quadratic equations. Application to symmetric function. Polynomial functions of 3^{rd} and 4^{th} degrees that can be reduced to quadratic ones.	Master the determination of roots by taking some concrete examples of quadratic equations. Also learn to determine the range of variables in a quadratic expression under a given condition.	

14.	**Partial Fractions** Types of partial fractions. Applications of partial fractions in summation of series and expansion of rational functions.	Master the techniques to resolve functions such as $\frac{A}{(x-\alpha)(x+\beta)}$, $\frac{A}{(x^2+bx+c)}$, $\frac{A}{(x+\alpha)(x^2+bx+c)}$, etc.	
15.	**Circular Measure**	Functions and their graphs. Odd, even and periodic functions. Trigonometric ratios of angles of any magnitude. Inverse trigonometric functions. Graphs of trigonometric functions. Definitions of a radian. Relationship between radians and degrees.	Construct examples of trigonometric functions and determine the periods, amplitude, phase etc. Use the formula $S = r\theta$ and $A = \frac{1}{2} r^2\theta$ in solving problems covering the arc length and sector area of a circle.
16.	**Compound Angle Formulae, Trigonometric Equations**	The formulae sin (A ± B), cos (A ± B), tan (A ± B) and their proofs. Multiple and half angles. Simple identities. The solution of simple trigonometric equations e.g. $a\cos\theta \pm b\sin\theta =$ $R\cos(\theta \pm \alpha)$. Factor Formula.	Master the methods of proof involving half and multiple angles in particular. Also work out various examples of trigonometric equations.
17.	**Sine and Cosine Rules**	Application of Sine and Cosine rules to the solution of triangles, heights and distances.	Master various methods of solutions of triangles excluding ambiguous cases.

18.	**Co-Ordinate Geometry** a) Distance between points, equations of a straight line. Solve problems involving straight lines in coordinate geometry. b) Conic Sections. i. find equations of circles, parabola, ellipse and hyperbola. ii. Find centre, foci directrices and eccentricity. iv. Sketch diagrams given equation of any of the conic section	Cartesian coordinates, distances between two points. The mid-point of a line segment. The gradient of a line. The equation of a line. Determination of Linear Law from appropriate data. The relationship between the gradients of a parallel and perpendicular line. (i). Equation of circles with radius r; centre (h,k) i.e. $(x-h)^2 + (y-k)^2 = r^2$ (ii). Equation of parabola, ellipse and hyperbola. (iii). Sketching of shapes of the conic sections from the equations. (iv). Determination of eccentricity, foci and directrices. (v). Finding equations of tangents and normal to the curves of the conic sections using the idea of differential coefficients.	Interpreted and use Linear equations particularly the forms $y = mx+c$ and $y-y_1 = m(x-x_1)$. Only simple and direct cases of finding equations and determination of centre, radius, foci, eccentricity, directrices from given equations of the conic sections should be treated.
19	**Complex Numbers** Perform arithmetic operations with complex numbers	The complex number system. Addition, subtraction, multiplication and division of surds and complex numbers. Modulus, conjugation, argument and phasor in the form (at, ab). Geometric interpretation. Polar representation. Nth roots of unity.	Construct various examples of complex numbers to find their magnitudes and arguments. To determine nth roots of a given complex equality e.g. $(1+i)^{\frac{1}{5}}$, $i^{1/5}$
20	**Graphs of Linear Relation:** Plot graph of linear relationship and apply to solve business problems	Cartesian plane. Table of values for linear relations. Graphs of linear relations. Simultaneous equations using graph. Using graph to show break – even point, total cost, total sales and margin of safety, etc.	Day-to-day business activities should be used as examples.

PAPER 2

S/N	TOPICS/OBJECTIVES	CONTENTS	ACTIVITIES/REMARKS
1.	**Limit and Continuity of Functions**	Definition of limit and continuity of functions with simple examples. Proof of $\frac{\sin\theta}{\theta} = 1$ as θ approaches zero. Asymptotes (parallel to the axes only) in graph sketching. Graphs of algebraic functions. (Polynomials and simple rational functions), trigonometric functions, exponential and logarithmic functions to various bases. Knowledge of the series expansion of e^x for all x and ln (1+ x) for $-1 < x \leq 1$	Construct examples of limits and test continuity at a given point. No analytic proofs are needed.
2.	**Differentiation of Algebraic Functions**	Limiting value of a function. The Gradient function. The Derivative of a function. Differentiation from first principle. Differentiation using formula $\frac{dy}{dx} = anx^{n-1}$ where y = ax^{n-1} for integer n. The derivative of a sum. Function of a function. The derivative of a product, quotient. Implicit differentiation. Application of differentiation laws.	The trainee should be led to apply differentiation laws to solve problems in minima and maxima, velocity, acceleration and rate of change. Derivatives of trig. functions. Inverse function, logarithms function. Exponential functions and higher derivatives may be introduced to the trainees.
3.	**Integration of Algebraic Functions** Integrate simple algebraic functions and apply them in solving problems involving area under the curve.	Definite integral and its representation as an area. Integration as the inverse of differentiation. Integration of elementary functions. Techniques of integration (by partial fractions, by substitution and by parts). Integration using identities and standard formulae. Applications of integration to areas and volumes.	Master various methods of integration. Application of definite integral to determine area under the curves for simple cases. Only proper integral need to be treated.
4.	**Differential equations**	First order differential equations only	Construct some simple examples of first order differential equations and integrate them. Only intuitive understanding of the concept needs to be given.
5.	**Vectors** Apply the triangular and	Scalars and Vectors – magnitude of vectors. The zero vector. The negative vector.	

	parallelogram laws of vectors to solve relevant problems in forces, velocity, wind, aeronautics etc. Relative velocity and acceleration.	Equality of vectors. Addition and subtraction of vectors – the triangular and parallelogram vectors. Components of a vector in two/three dimensions.	
6.	**Regression**	Scatter diagrams. Regression line and its characteristics. Linear regression equation and curves. Fitting of regression lines by the method of least squares. The meaning of regression coefficient and its estimation from graph. The use of regression lines.	Some simple concrete examples should be constructed. No exponential or multiple regression is required.
7.	**Correlation Coefficient**	Product moment correlation coefficient and Spearman's rank correlation coefficients.	Simple examples of these coefficients should be constructed.
8.	**Averages** Solve problems involving averages.	Meaning of moving averages. Calculation of moving averages orders, 1,2,3,4,5. Estimation of trend from moving averages. Mean, median, mode and variance.	Only the trend – movement or moving average should be treated, cycled or irregular pattern should be discarded. Grouped and ungrouped data should be treated.
9.	**Determinants and Matrices** Definition and properties of second and third order determinants. Application of determinants to solve simultaneous linear equations using Cramer's rule. Algebraic operations, addition, subtraction and multiplication of a matrix by a scalar. Restricted to 3 x 3 matrices.	Need to work-out several concrete examples of determinants and matrices. Not to go beyond Cramer's rule.	
10.	**Binomial** Solve problems using the Binomial expansion formula.	Pascal's triangle, Binomial Expansion formula.	Knowledge of permutation and combination required.