

**NATIONAL BUSINESS AND TECHNICAL EXAMINATIONS BOARD
MAY/JUNE 2006 NBC/NTC EXAMINATION
MATHEMATICS**

- 1(a) Evaluate $\frac{0.0285 \times 0.267}{3.36}$, leaving your answer in standard form.
- (b) If the angles of a polygon are given as x° , $(x+20)^\circ$, $(2x-10)^\circ$, $(3x+10)^\circ$ and $(x-10)^\circ$. Find the value of x .

Solution

(a)
$$\frac{0.0285 \times 0.267}{3.36}$$

$$= \frac{0.0076095}{3.36}$$

$$= 0.0022647 \approx 0.002265 \text{ in standard form we have it as } 2.265 \times 10^{-3}$$

(b) sum of angles = 540°
 $x + x + 20 + 2x - 10 + 3x + 10 + x - 10 = 540^\circ$
 $\therefore x = 66.25^\circ$

ALITER for 1(b)

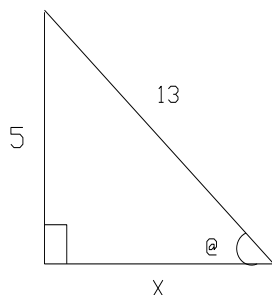
The corresponding exterior angles are $(180 - x)^\circ$, $(160 - x)^\circ$, $(190 - 2x)^\circ$, $(170 - 3x)^\circ$, $(190 - x)^\circ$

Equating sum of the exterior angles to 360° , we have $x = 66.25^\circ$

- 2(a) If the angles of a pentagon are x° , x° , $2x^\circ$, $(2x+40)^\circ$, $(2x+10)^\circ$. Find the value of the biggest angle.
- (b) Given that $0^\circ < \theta < 90^\circ$ and $\sin \theta = 5/13$, Find without using tables or calculator $\cot^2 \theta$

Solution

Sum of angles of a pentagon is 540°
 $\therefore x + x + 2x + 2x + 40 + 2x + 10 = 540^\circ$
 Therefore $x = 61.25^\circ$



2(b)

By Pythagoras' theorem

$$x = \sqrt{13^2 - 5^2} = 12$$

$$\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\tan^2\theta} = \frac{\text{Adjacent}}{\text{Opposite}}$$

$$\therefore \cot^2\theta = \frac{12^2}{5^2} = \frac{144}{25} = 5\frac{19}{25}$$

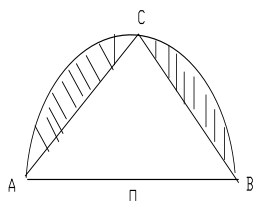
ALITER

$$\cot^2\theta + 1 = \operatorname{Cosec}^2\theta \text{ or equivalent}$$

$$\operatorname{Cosec}^2\theta = \frac{169}{25}$$

$$\therefore \cot^2\theta = 5\frac{19}{25}$$

- 3(a) The 1st and 5th terms of a G.P are 81 and 1 respectively. Determine the three terms between the two numbers.



In the figure AB is the diameter of a semi-circle, AOCB. If /AB/ = 10cm and /AC/ = 8cm. Calculate the area of the shaded portion.

Solution

(a) $a = 81$ and $ar^{5-1} = ar^4 = 1$

Solving the equation, we have $r^4 = \frac{1}{81}$

$\therefore r = 1/3$

Using ar^{n-1} , we have between 81 and 1, $n=2, 3$ and 4; 27, 9 and 3

b) Area of a circle = $\pi r^2 = \frac{22}{7} \times 5^2 = 78.55\text{cm}^2$

$$\begin{aligned} & \qquad \qquad \qquad 7 \\ \therefore \text{Area of semicircle} &= 39.275\text{cm}^2 \\ \text{Area of triangle} &= \frac{1}{2}bh \\ \text{Height } h &= 9.6\text{cm} \\ \therefore \frac{1}{2} \times 5\text{cm} \times 9.6\text{cm} &= 24\text{cm}^2 \end{aligned}$$

- 4(a) Solve for t in the equation, $\frac{t}{4} + 6 = 2\frac{1}{3}t$
- (b) The dimensions of a room are 15m long, 8m wide and 10m high. Calculate the angle which the diagonal of the room makes with the floor.

Solution

(a) $\frac{t}{4} + 6 = 2\frac{1}{3}t$, collecting like terms, we have $2\frac{1}{3}t - \frac{t}{4} = 6$

$$\therefore t = 2\frac{22}{25}$$

ALITER

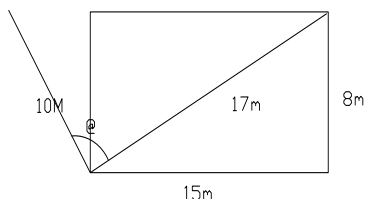
Solving and clearing fractions, $\frac{t}{4} + 6 = \frac{7}{3}t$

$$\therefore t = 2\frac{22}{25}$$

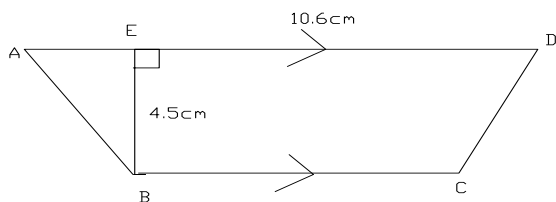
- (a) the diagonal of the floor of the room = 17m; by Pythagoras' theorem.

$$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{10}{17} = 0.5882$$

$$\therefore \tan \theta = 0.5332 \text{ and } \theta = 30.47^\circ$$



5(a) Find the sum of the first 100 even numbers



(b) In the figure $AD \parallel BC$, BE is perpendicular to AD . If the area of $ABCD$ is 40.5cm^2 , find the length of BC .

Solution

5(a) Even numbers, $a = 2$, $d = 2$, $n = 100$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{100}{2}(4 + (99) \times 2)$$

$$= 10,100$$

(b) Total Area = $\frac{1}{2}(4.5)(10.6 + BC) = 40.5$
 Solving for BC , we have
 $BC = 7.4\text{cm}$

- 6(a) Given that $p = \frac{r}{r^2 - 4}$, $q = \frac{r}{r + 2}$, express $2q + 8p$ in terms of r in the simplest form.
- (b) Solve the equation $3x^2 + 4x - 7 = 0$, leaving your answer to 2 decimal places.
- (c) Simplify: $\log 6 + \frac{1}{2} \log 81 - \log 27$

Solution

- (a) Substituting the values of p , q and taking L.C.M.

$$\begin{aligned} 2p + 8q &= \frac{2r(r-2) + 8r}{(r+2)(r-2)} \\ &= \frac{2r}{r-2} \end{aligned}$$

- (b) $(3x+7)(x-1) = 0$
 $3x+7=0$ or $x-1 = 0$
 either $x = -2.333$ or $x = 1$

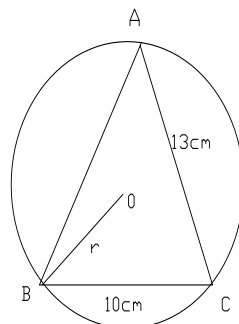
ALITER

$$x = \frac{-4 \pm \sqrt{16 + 4(3)(7)}}{2(3)}$$

either $x = -2.33$ or $x = 1$.

- (c) $\log 6 + \frac{1}{2} \log 81 - \log 27$; NB: $\log 81^{\frac{1}{2}} \equiv \log \sqrt{81}$
- $$\begin{aligned} &= \log 6 + \log \sqrt{81} - \log 27 \\ &= \log 6 + \log 9 - \log 27 \\ &= \log \left(\frac{6 \times 9}{27} \right) \\ &= \log \left(\frac{54}{27} \right) \\ &= \log 2 \end{aligned}$$

- 7(a) The difference between the reciprocal of a number and the reciprocal of the sum of 3 and the number is $\frac{3}{40}$. What is the number?

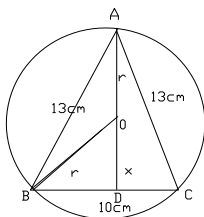


(b)
 In the figure given ABC is an isosceles triangle inscribed in a circle centre O. If $\angle AB = \angle AC = 13\text{cm}$ and $\angle BC = 10\text{cm}$. Calculate the radius of the circle to the nearest whole number.

Solution

(a) Let the number be x. From the question we have: $\frac{1}{x} - \frac{1}{x+3} = \frac{3}{40}$

Simplifying, we obtain $x^2 + 3x - 40 = 0$
 $\therefore x = -8$ or $x = 5$



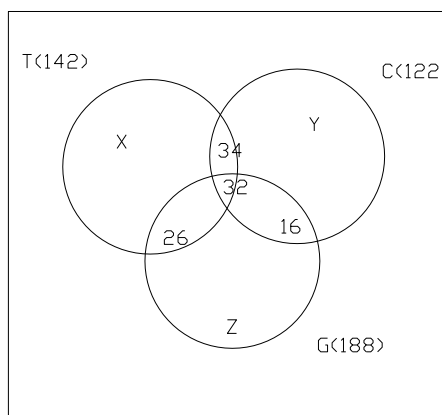
(b)
 $r^2 = x^2 + 5^2$ (i)
 $\therefore AD = \sqrt{13^2 - 5^2} = 12\text{cm}$
 $x = 12 - r$ (ii)

Solving for r: $r^2 = (12-r)^2 + 5^2$
 $r = 7.042$
 $\therefore r = 7.0 \text{ cm}$

- 8(a) In an opinion survey, a group of people were interviewed about the daily newspaper read by them. 142 read the Tribune, 122 read the Champion and 118 read the Guardian. 66 read both the Tribune and the Champion. 48 read the Champion and the Guardian while 58 read the Tribune and the Guardian. 32 read all the three newspapers.
- Illustrate the information using a Venn diagram.
 - How many people were interviewed?
 - How many people read the Guardian only?

Solution

(i)



- $$x + 26 + 34 + 32 = 142$$

$$\therefore x = 50$$

$$y + 34 + 16 + 32 = 122$$

$$\therefore y = 40$$

$$z + 26 + 32 + 16 = 118$$

$$\therefore z = 44$$

Total number of people that were interviewed = $50 + 40 + 44 + 26 + 34 + 16 + 32 = 242$
- $$z + 26 + 32 + 16 = 118$$

$$\therefore z = 44 \text{ (read Guardian only)}$$

9(a) Complete the table of values for the relation $y = 2 + x - x^2$ for $-3 \leq x \leq 4$

x	-3	-2	-1	0	1	2	3	4
y		-4		2				

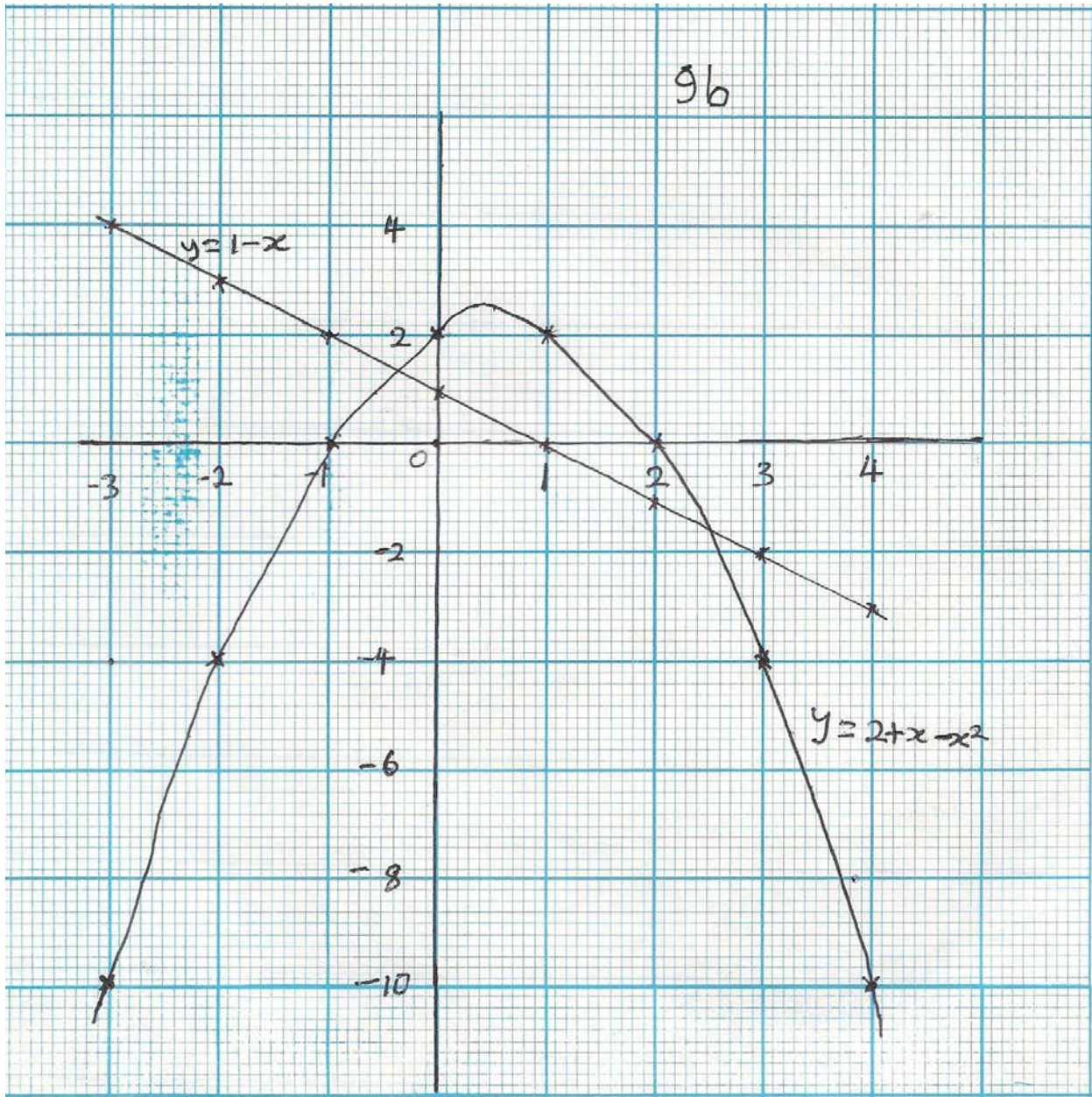
- Using a scale of 2cm to 1 unit on the x-axis and 2 units along the y-axis, draw the graph of $y = 2 + x - x^2$ with your values in 9(a)
- From your graph

- (i) Find the co-ordinates of the point at which y is greatest.
- (ii) Estimate the roots of the equation $2 + x - x^2 = 0$
- (d) Using the same scale and axes, draw the graph of $y = 1 - x$
- (e) Use your graphs to solve the equation $1 + 2x - x^2 = 0$

Solution

(a)

x	-3	-2	-1	0	1	2	3	4
y	-10	-4	0	2	2	0	-4	-10



- (c) (i) The co-ordinates = $(0.5, 2.3) \pm 0.1$
 (ii) The roots $x = -1$ or $x = 2 \pm 0.1$

- (d) The graph of $y = 1 - x$
 Table of $y = 1 - x$

x	-3	-2	-1	0	1	2	3	4
y	4	3	2	1	0	-1	-2	-3

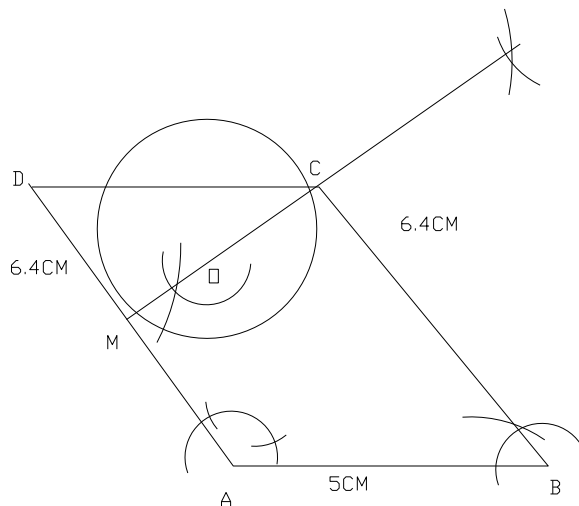
- (e) From the graph, the roots of $1 + 2x - x^2 = 0$ are $x = -0.4 \pm 0.1$, and $x = 2.4 \pm 0.1$

- 10(a) With a pair of compasses and ruler only, construct parallelogram ABCD such that $\angle A = 50^\circ$, $\angle B = 60^\circ$ and $\angle C = 70^\circ$

- (i) Bisect $\angle C$ and let the bisector meet AD at M.
 (ii) Draw a circle centre O with CM as diameter.
 (iii) Measure $\angle ODM$ and $\angle OCM$.

- (b) Given two points X (lat. $50^\circ N$, long $85^\circ E$) and Y (lat. $50^\circ N$, long $25^\circ E$). Find the distance between them along the common parallel of latitude. Take R to be 6400km. π to be 3.14 and correct your answer to 2 decimals.

Solution



- (a)
 (b) The distance between the two points, difference in longitude = 60°

$$= \frac{60^\circ}{360} \times 2\pi \times 6400 \times \cos 50^\circ$$

$$= 4305.90 \text{ km}$$

11(a) The cost of sewing a set of suit is partly constant and partly varies inversely with the number of days it takes to sew the suit. If the suit takes 3 days to sew, the cost is ₦3,000. If it takes 5 days, the cost is ₦2,500. Find the cost if the tailor takes 4 days to sew a particular set of suit.

(b) Calculate the perimeter of the major segment of a circle radius 17.5cm determined at the centre of the circle. Take $\pi = \frac{22}{7}$

Solution

We can represent the question with the following equations:

$$3a + b = 900$$

$$5a + b = 12,500$$

$$\therefore a = 1750$$

$$\text{and } b = 3750$$

$$\text{The law of the variation, } C = 1750 + \frac{3750}{D}$$

$$\text{For 4days, } D = 4,$$

$$C = 1750 + 937.5$$

$$= \text{N } 2687.50$$

(b) Perimeter of a segment of a circle = length of arc + length of chord

$$\text{length of the semi circular arc} = \frac{22}{7} \times \frac{35}{2} = 55\text{cm}$$

$$\text{length of chord} = 17.5\text{cm} + 17.5\text{cm} = 35\text{cm}$$

$$\therefore \text{perimeter} = 55\text{cm} + 35\text{cm} = 90\text{cm}.$$

12(a) Mention two uses of averages

(b) List two merits and two demerits of the

(i) mean, and

(ii) median, as measures of central tendency.

(c) Calculate

(i) range, and

(ii) standard deviation of the set of data: 4,5,8,3,9,10,15,12,8,16

Solution

12(a) Two (2) uses of average

(b) Listing of two (2) merits and two (2) demerits of mean and median.

- (c) (i) range = 16 - 3 = 13
 (ii) S. D = $\sum (x - \bar{x})^2 = 174$

$$\text{S.D.} = \sum \left(\frac{x - \bar{x}}{N} \right)^2 = \frac{174}{10} = 17.4$$

$$\text{S.D.} = \sqrt{17.4} = 4.17$$

- 13(a) A man saves a sum of money in a bank which pays simple interest at the rate of 4% per annum. If after 3 years the money amounts to ₦20,000.00. Find the sum invested.
- (b) Three boys did a business together and shared the profit they made. The first got 20% of the share and got 37½ % of the remainder. If the third boy received ₦450.00, how much was the profit they shared?

Solution

(a) $I = \frac{PRT}{100} = \frac{3P}{25}$

$$\text{Amount} = A = P + \frac{3p}{25}$$

$$\text{sum invested} = P + \frac{3p}{25} = 20,000$$

$$\therefore P = \text{₦}17857.14$$

(b) remainder after the first boy's share = 100% - 20% = 80% $\Rightarrow \frac{80}{100} = \frac{4}{5}$

$$\text{The second boy's share} = \frac{75}{2} \% \times \frac{4}{5}$$

$$= \frac{300}{10} = 30 \Rightarrow \frac{30}{100} = \frac{3}{10}$$

Proportion left for the third boy = ½.

If the third boy received ₦450, therefore the total profit shared would be
~~₦450~~ + ~~₦450~~ = ~~₦900.00~~

- 14(a) A man's total salary for 4 months in a particular year was ₦14,001.00. Then there was salary review in which monthly salaries were doubled. Calculate his total salary in that year.
- (b) Ojo, Obi and Bala are three partners who own a business venture. It was agreed that the profit will be shared such that Ojo received $\frac{2}{5}$. Obi receives $\frac{2}{3}$ and Bala receives $\frac{1}{4}$ of the total profit made. If at the end of the year Ojo receives ₦3,200 less than Obi, what is Bala's share of the total profit?

Solution

14(a) Monthly pay before salary review = $\frac{\text{₦}14001}{4}$

= ₦3,500.25

monthly salary after review = ₦3,500.25 x 2

= ₦7000.50

8 months salary after review = ₦7000.50 x 8

= ₦56004.00

Total salary for the year = ₦56004 + ₦14001

= ₦70,005.00

(b) Let x represent the total profit. Therefore we have the equation $\frac{2x}{5} + 3200 = \frac{2x}{3}$

$$= \frac{2x}{5} - \frac{2x}{3} = -3200$$

L.C.M approach - $\frac{4x}{15} = -3200$

$\therefore x = \text{₦}12,000$

then Bala's share will be $\frac{1}{4}$ of ₦12,000 = ₦3,000.00

- 15(a) A man bought 10,000 ₦1.00 shares of FELGRA PLC at ₦1.50 per share through a broker. If the brokerage commission rate is 5.4% plus ₦300 for any transaction of ₦5,000 or more, calculate the:
- broker's commission and
 - amount the man received if a dividend of 15% was declared.
- (b) A man is to pay back a loan of ₦48,000 monthly for 3 years. If the amount is borrowed at the rate of $12\frac{1}{2}\%$ per annum, what will be his monthly installment payment?

Solution

(a) (i) Market value of the shares = ₦1.50 x 10,000 = ₦15,000

$$\text{Broker's commission} = \left(\frac{\text{₦}15,000}{100} \times 5.4 \right) + \text{₦}300$$

$$= \text{N}810 + \text{N}300$$

$$= \text{N}1110.00$$

(ii) Amount the man received if a dividend of 15% was declared.

Nominal value of the share = ~~N~~10,000

Dividend received ~~N~~1500.00

(b) $I = \frac{PTR}{100}$

$$\text{Interest on the loan} = \frac{\text{N}48,000 \times 3 \times 12\frac{1}{2}}{100} = \text{N}18,000$$

$$\text{total money owed} = \text{N}48,000 + \text{N}18,000 = \text{N}66,000$$

the monthly installment will be $\frac{\text{N}66,000}{36 \text{ months}}$

$$= \text{N}1833.33$$